4190.417: Computer Animation

Midterm Exam (October 19, 2005)

1	/10	**5**	/20
2	/ 5	**6**	/10
3	/15	**7**	/20
4	/20	Total	/100

Name: ____________________

ID Number: ____________________

1. (10 points) Let $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are affine spaces. Show that if $F : \mathcal{A} \to \mathcal{B}$ and $G : \mathcal{B} \to \mathcal{C}$ are affine maps, then the composed map $H = G \circ F : \mathcal{A} \to \mathcal{C}$ is also an affine map.

2. (5 points) Discuss the pros and cons of unit quaternions in comparison with homogeneous matrices.

3. (15 points) Consider a camera held by a platform consisting of three gimbles. The outer-most gimble allows rotation about the z axis and the middle and inner-most gimbles allows rotations about the x and the y axes, respectively. The rotation angles about z, x, and y axes are denoted by a, b, and c, respectively.

3-1. Represent the orientation of the camera in unit quaternions.

3-2. Represent the orientation of the camera in rotation matrices.

3-3. When do the gimbles yield "gimble lock"? Describe the angles specifically and depict the gimble lock situation.

4. (20 points) We have key poses of an articulated figure and want to produce a smooth motion interpolating those key poses using cubic polynomial splines. Answer the following questions.

4-1. What is the difference between natural, Catmull-Rom, and B-splines considering the continuity and local controllability of splines ?

4-2. Why do we prefer splines over polynomial interpolation?

4-3. Why do we need two re-parameterizations (arc length and speed control) for the splines?

4-4. Do you think that the convex hull property of splines is important for keyframing articulated figures? Explain your answer.

5. (20 points) Answer the following questions about solving systems of linear equations.

5-1. What is the difference between direct and iterative methods?

5-2. What is the advantages of LU decomposition over Gauss elimination?

5-3. Consider $A\mathbf{x} = \mathbf{b}$ as a linear mapping from the vector space $\mathbf{x}$ to the vector space $\mathbf{b}$. What are the range and the null space?

5-4. How do you compute the orthogonal basis vectors of the range and the null space using SVD?

6. (10 points) Show that every non-singular affine transformation of the affine plane is the composite of one scale, one translation, one rotation, and one shear. A scale transformation is one where $\mathcal{O} \to \mathcal{O}, \vec{v}_1 \to a\vec{v}_1$, and $\vec{v}_2 \to b\vec{v}_2$, where $(\vec{v}_1, \vec{v}_2, \mathcal{O})$ is a frame, and where a and b are non-zero scalars. A shear is a transformation such that $\mathcal{O} \to \mathcal{O}$, $\vec{v}_1 \to \vec{v}_1$, $\vec{v}_2 \to a\vec{v}_1 + b\vec{v}_2$.

7. (20 points) Answer the following questions by "O" or "X" (2 points for each question and -1 for a wrong answer)

7-1. All plain curves that satisfy variation diminishing has the convex hull property. (   )

7-2. All 3-dimensional orientations can be represented by Euler angles. (   )

7-3. There exists a one-to-one mapping between unit quaternions and rotation matrices. (   )

7-4. Multi-linear interpolation is invariant under affine transformations. (   )

7-5. $\exp(v_1 + v_2) = \exp(v_1)\exp(v_2)$ for any $v_1, v_2 \in \mathbb{R}^3$ (   )

7-6. $\text{slerp}(t; q_1, q_2) = \text{slerp}(1 - t; q_2, q_1)$ for any $q_1, q_2 \in \mathbb{S}^3$ (   )

7-7. $v = \log(\exp(v))$ for any $v \in \mathbb{R}^3$ (   )

7-8. The object moving along a slerp has a constant angular velocity. (   )

7-9. Natural splines have the convex hull property. (   )

7-10. B-spline have the convex hull property. (   )