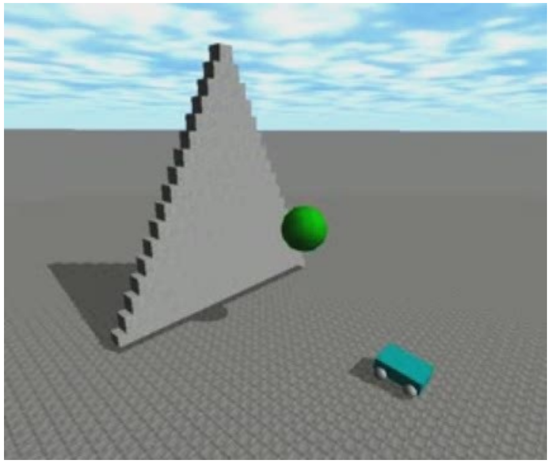
# Physics Simulation

Yoonsang Lee

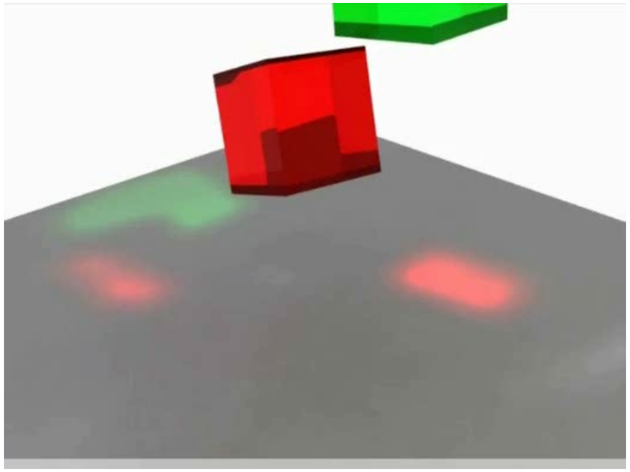
# Physics Simulation

- Simulate "Laws of Physics"

Rigid body



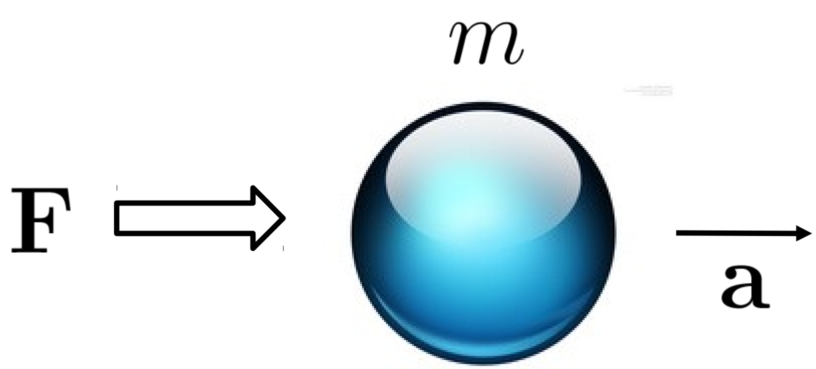
Soft body



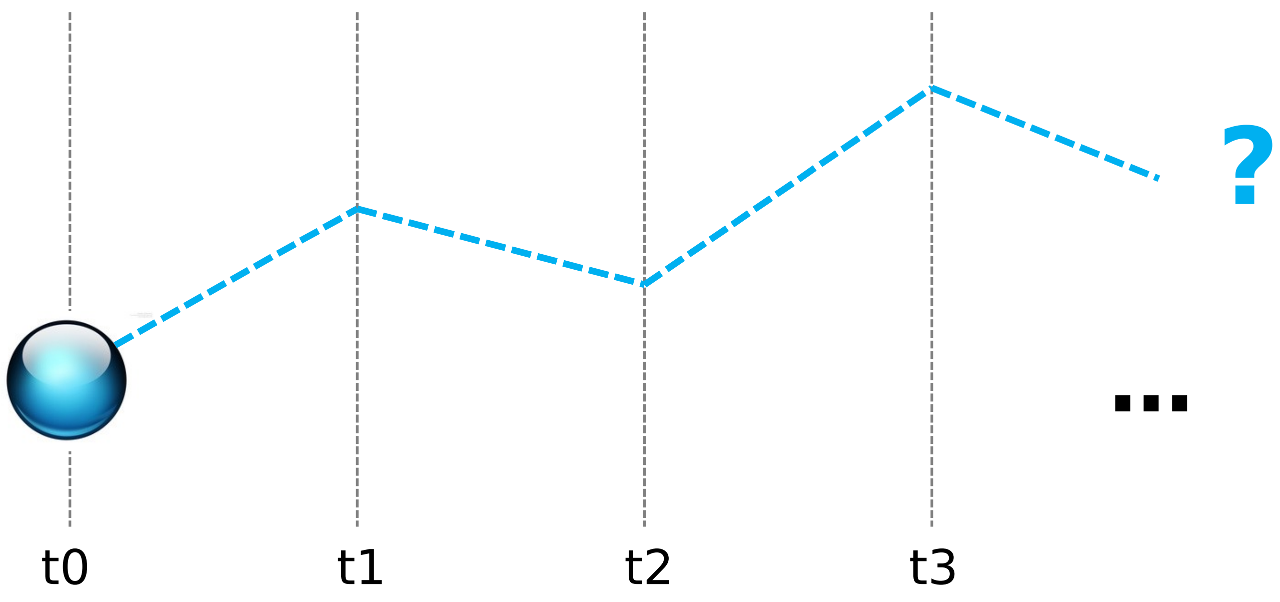
Fluid

# Equations of Motion

$$\mathbf{F} = m\mathbf{a}$$



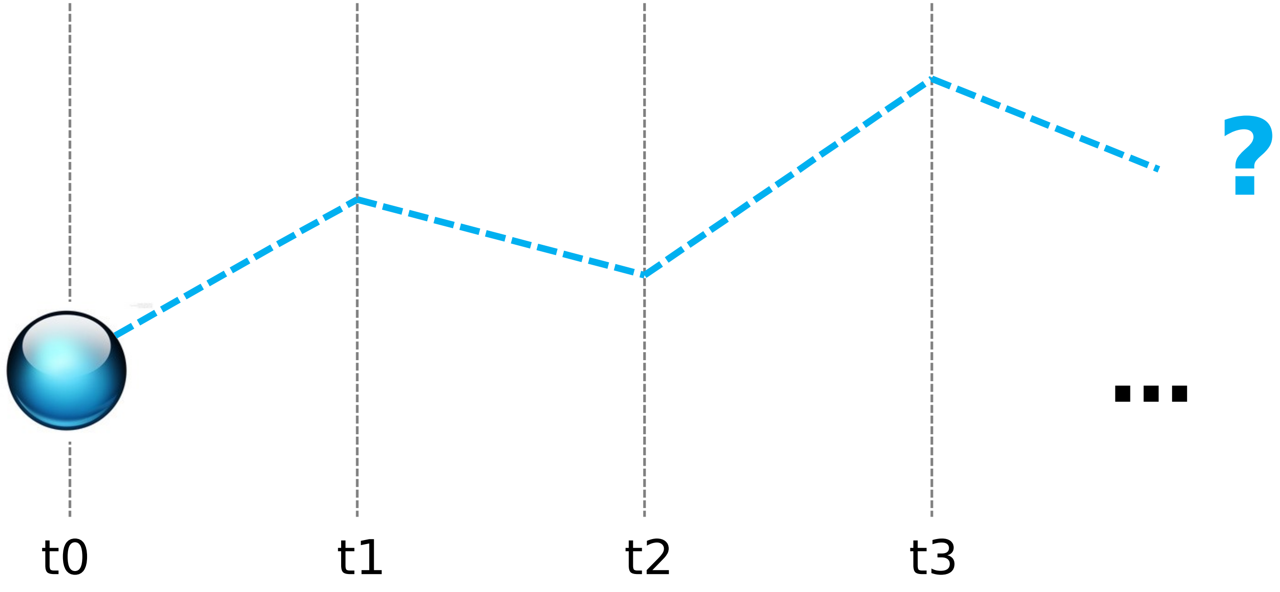
# Solving Equations of Motion



# Solving Equations of Motion

- State : $(\mathbf{x}(t), \mathbf{v}(t))$
- Given $(\mathbf{x}(t_0), \mathbf{v}(t_0))$, $\mathbf{a}(t) = \dfrac{\mathbf{F}(t)}{m}$

$$(\mathbf{x}(t_1), \mathbf{v}(t_1)), (\mathbf{x}(t_2), \mathbf{v}(t_2)), \ldots (\mathbf{x}(t_n)), \mathbf{v}(t_n)) = ?$$

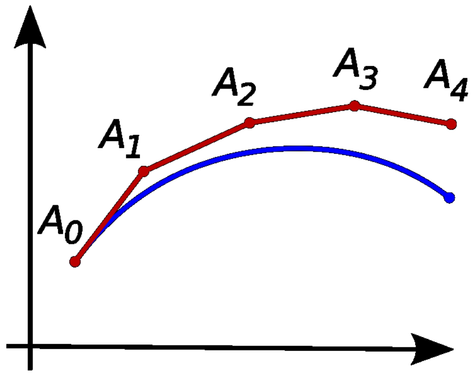


# Solving Equations of Motion

- Integration

$$\mathbf{x}(t+h) = \mathbf{x}(t) + h\dot{\mathbf{x}}(t)$$
$$\mathbf{v}(t+h) = \mathbf{v}(t) + h\dot{\mathbf{v}}(t)$$

(h : time-step)



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- Integration

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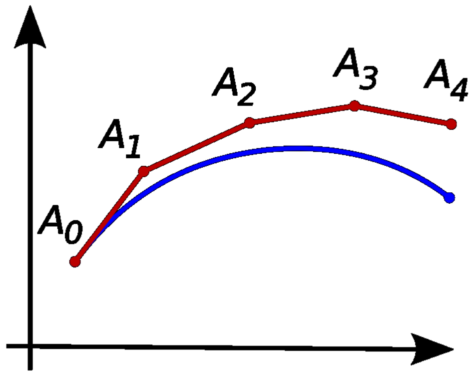
(h : time-step)

$$\dot{\mathbf{x}}(t) = \mathbf{v}(t)$$
$$\dot{\mathbf{v}}(t) = \mathbf{a}(t) = \frac{\mathbf{F}(t)}{m}$$



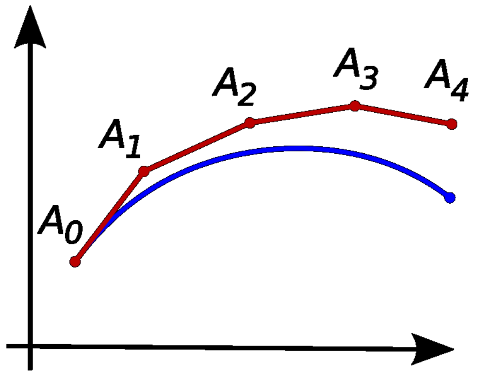
# Solving Equations of Motion

- Integration (Euler method)

$$\mathbf{x}(t+h) = \mathbf{x}(t) + h\dot{\mathbf{x}}(t)$$
$$\mathbf{v}(t+h) = \mathbf{v}(t) + h\dot{\mathbf{v}}(t)$$

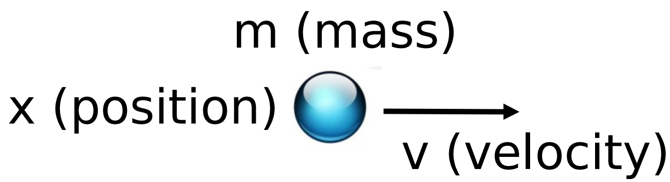
(h : time-step)

$$\dot{\mathbf{x}}(t) = \mathbf{v}(t)$$
$$\dot{\mathbf{v}}(t) = \mathbf{a}(t) = \frac{\mathbf{F}(t)}{m}$$




- Integrate method
  - Euler, Runge-Kutta, backward Euler(implicit)

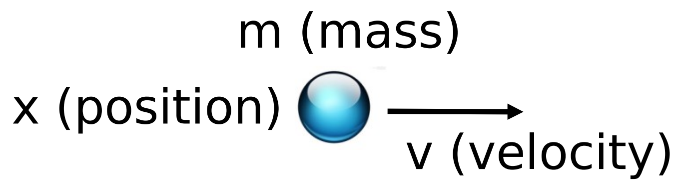
# Simple system : Particle System

- Particle : 

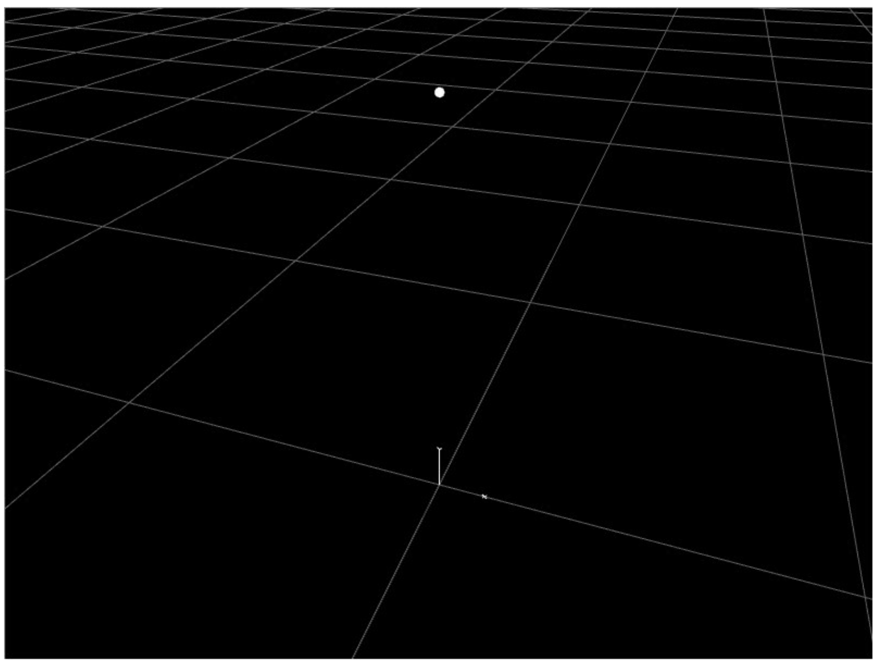
- $\mathbf{F} = m\mathbf{a}$

# Simple system : Particle System

- Particle :




- $\mathbf{F} = m\mathbf{a}$



(gravity only : F = -mg)

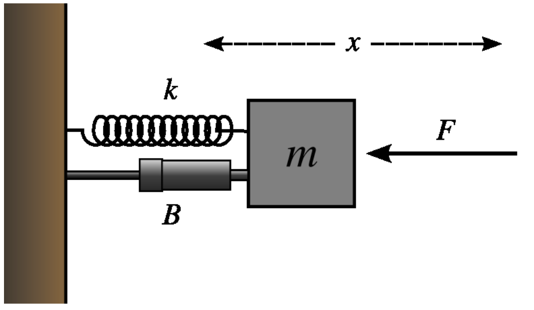
| | | |
|---|---|---|
| t:0.00 | x:1.00 | v:-0.33 |
| t:0.03 | x:0.99 | v:-0.65 |
| t:0.07 | x:0.97 | v:-0.98 |
| t:0.10 | x:0.93 | v:-1.31 |
| t:0.13 | x:0.89 | v:-1.63 |
| t:0.17 | x:0.84 | v:-1.96 |
| t:0.20 | x:0.77 | v:-2.29 |
| t:0.23 | x:0.70 | v:-2.61 |
| t:0.27 | x:0.61 | v:-2.94 |
| t:0.30 | x:0.51 | v:-3.27 |
| t:0.33 | x:0.40 | v:-3.59 |

…

# Particle System

- Spring force

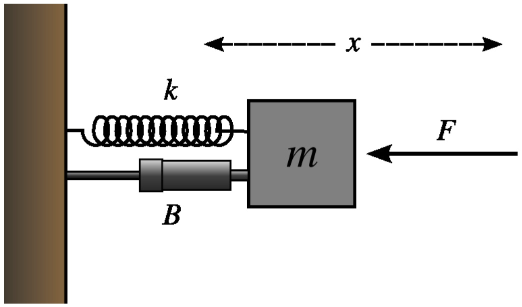
$$F_s = -kx + -c\dot{x}$$

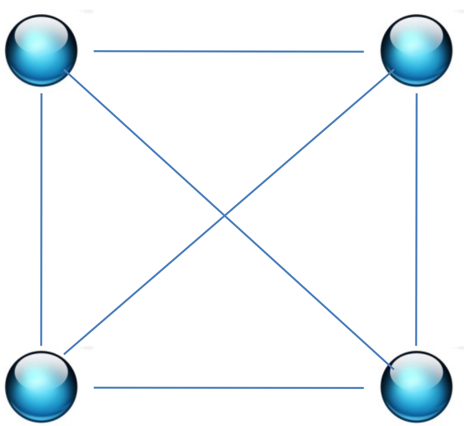


# Particle System

- Spring force

$$F_s = -kx + -c\dot{x}$$

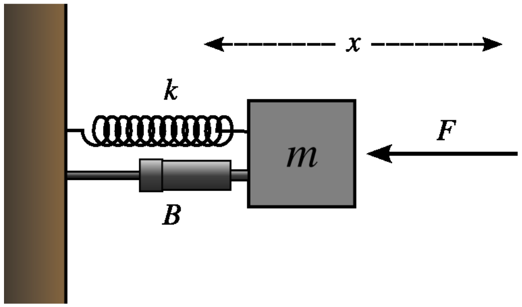


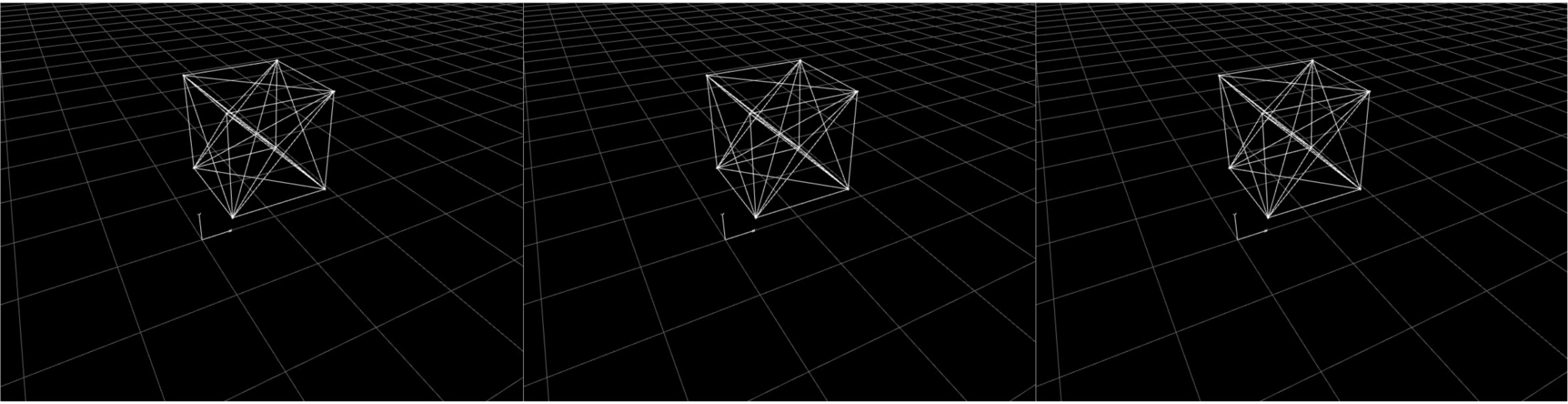



# Particle System

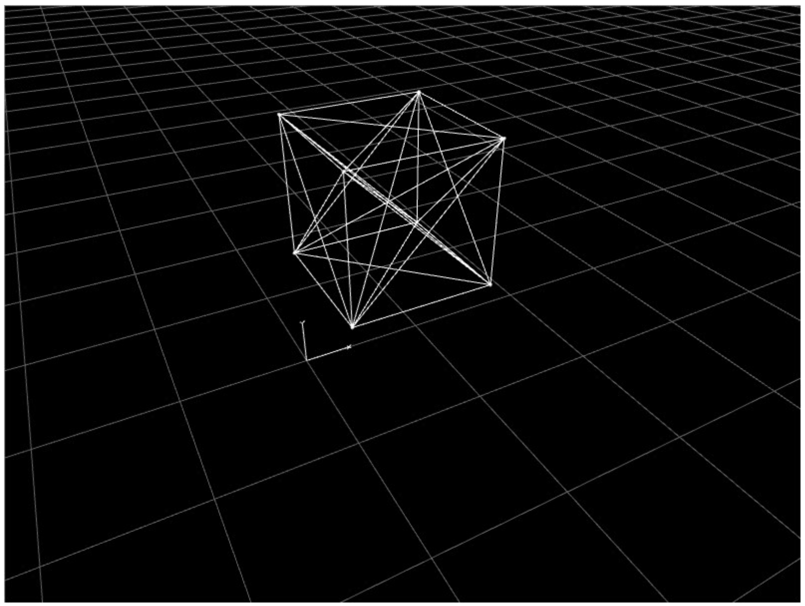
- Spring force

$$F_s = -kx + -c\dot{x}$$

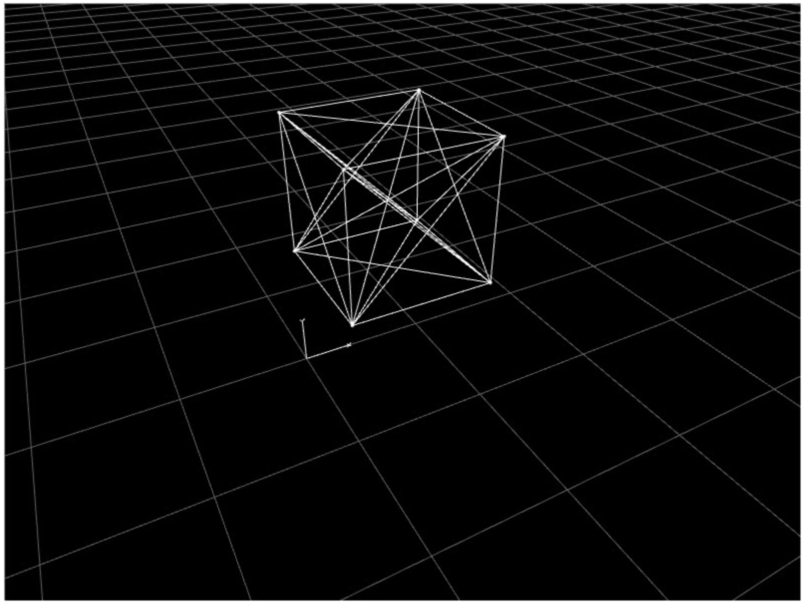


# Stability & Time-step Size

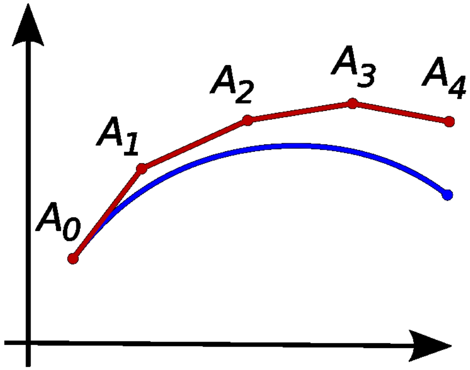


timestep = 0.003



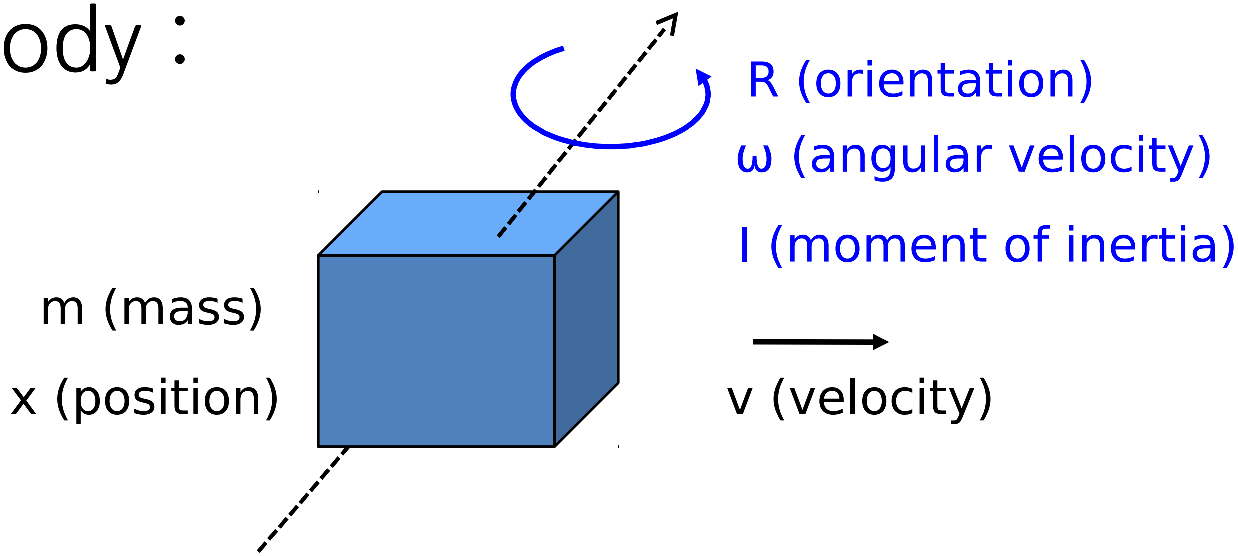
timestep = 0.033

- Too large step size -> diverges
- Too small step size -> very slow
- Depending on integration methods



# Rigid Body Dynamics

- Rigid body :




- $$\mathbf{F} = \frac{\mathrm{d}}{\mathrm{d}t}(m\mathbf{v}) = m\mathbf{a}$$

$$\tau = \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{I}\omega) = \dot{\mathbf{I}}\omega + \mathbf{I}\alpha$$

# Rigid Body Dynamics

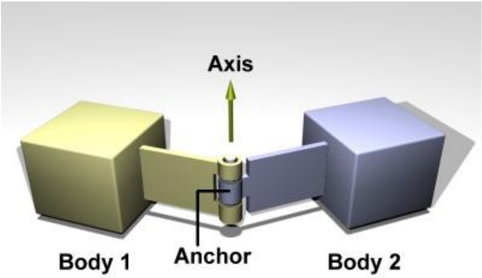
- State : $(\mathbf{x}(t), \mathbf{v}(t), \mathbf{R}(t), \omega(t))$
- Integration

  …

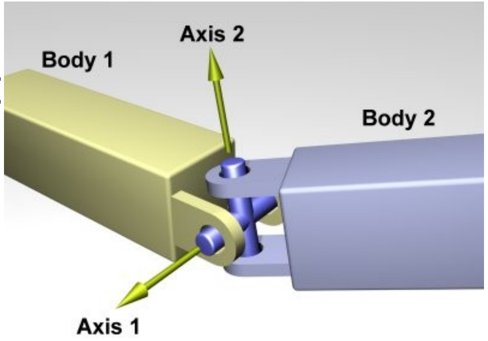
  - Usually used when rotational movement is more important than material deformation.

# Joint Constraint

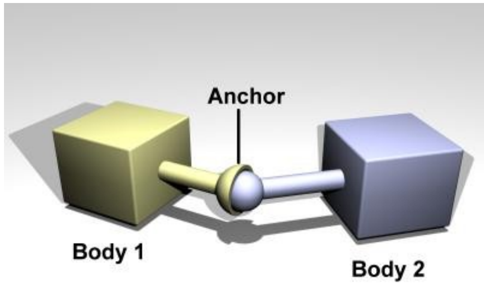
hinge joint
1 DOF



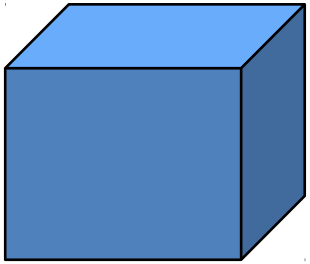

universal joint
2 DOF




ball joint
3 DOF

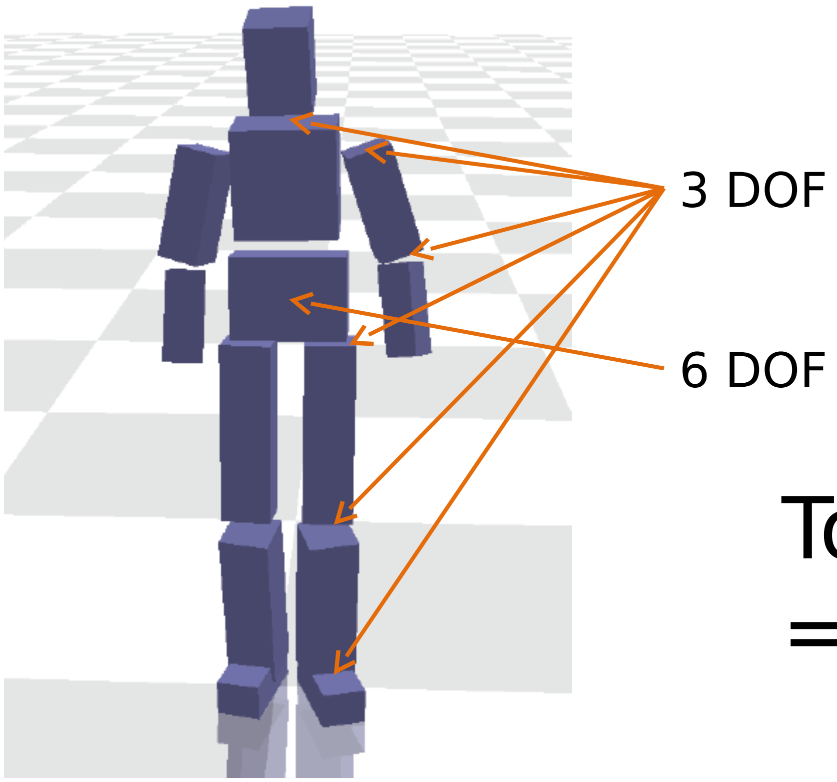



floating body
6 DOF



- Degrees of freedom (DOF) : number of independent variables tha t specify position (or orientation) of the system

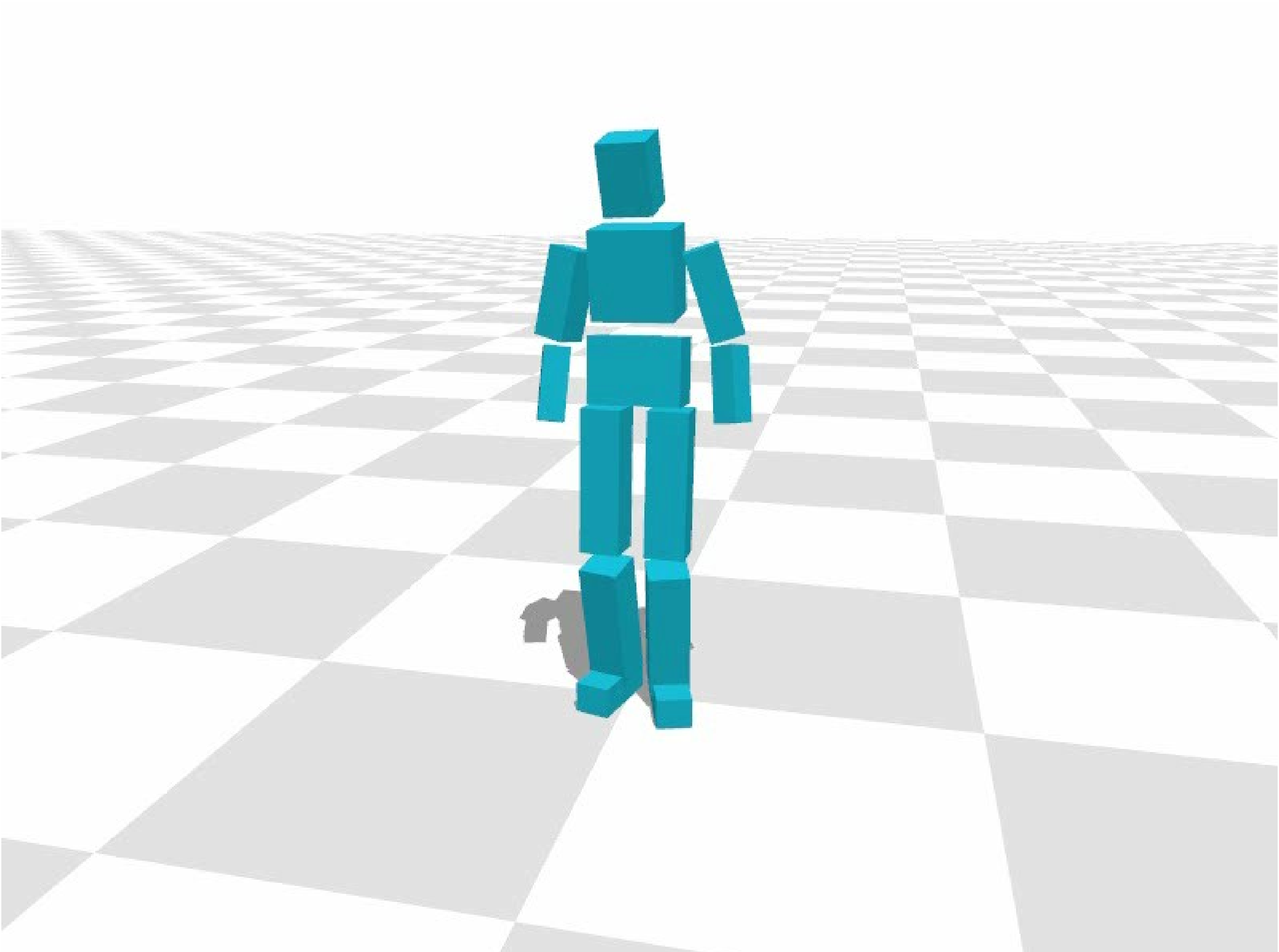
# Articulated Body




Total DOF of system
= 6 + 3*12 = 42

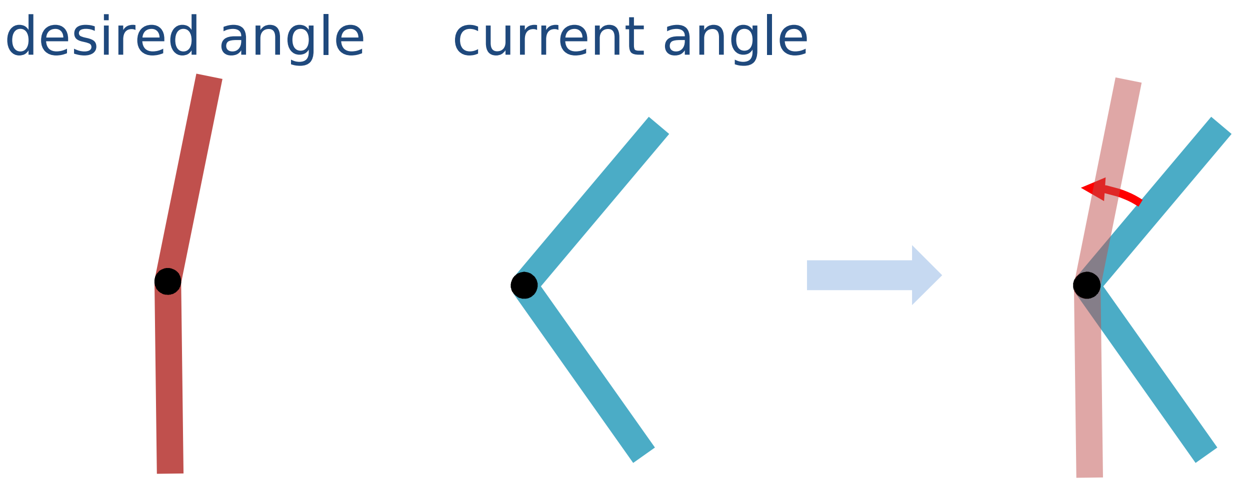
$$M\ddot{q}(t) = C(q,\dot{q}) + \tau(q) + F(q,\dot{q})$$

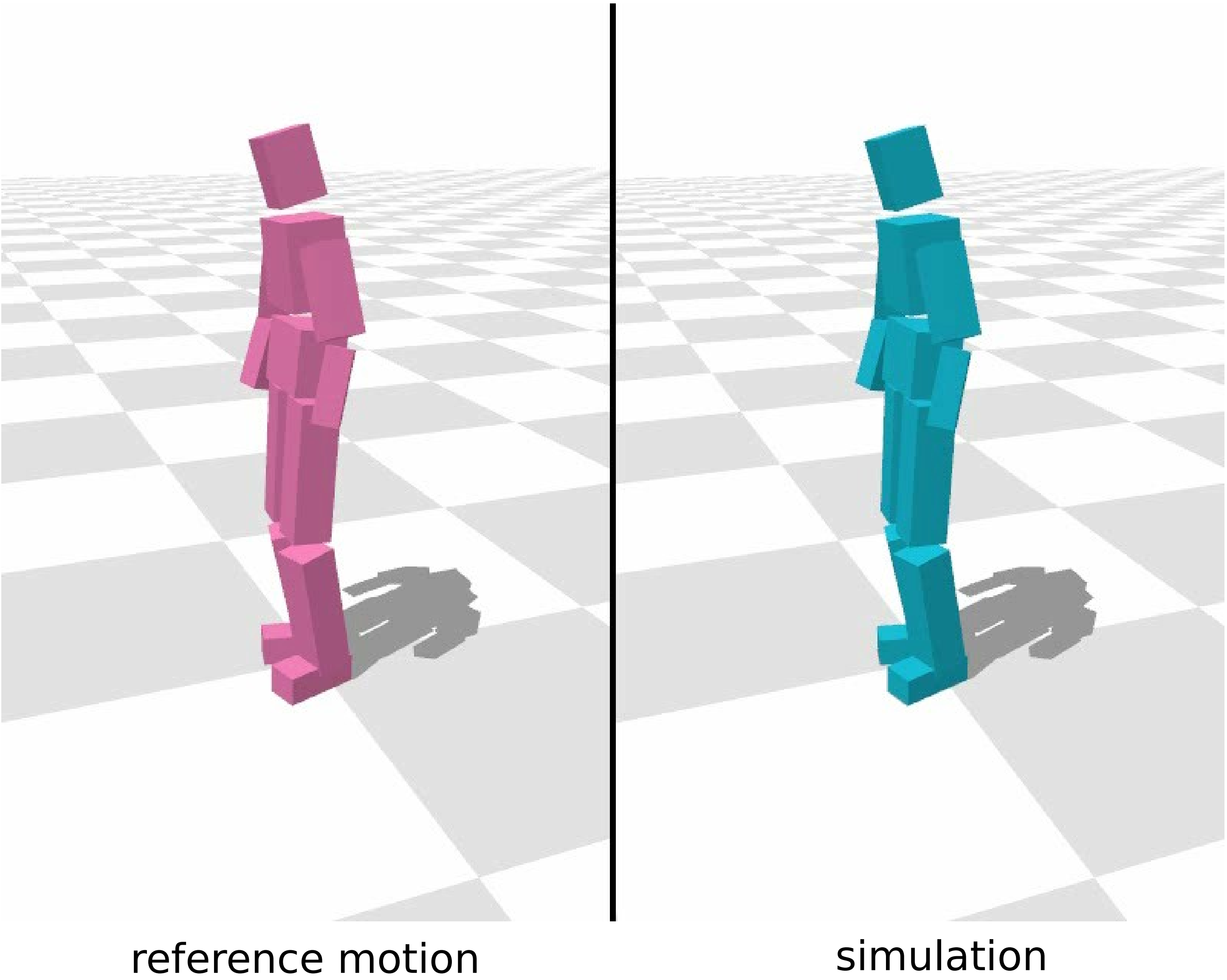
# Articulated Body



# Tracking Control

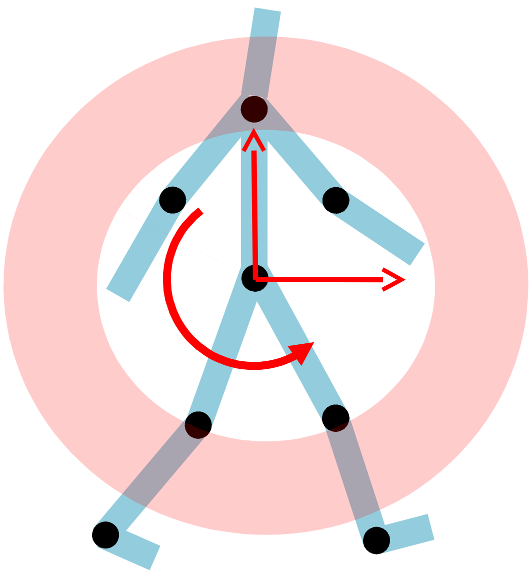
- Computing torques to follow reference data.

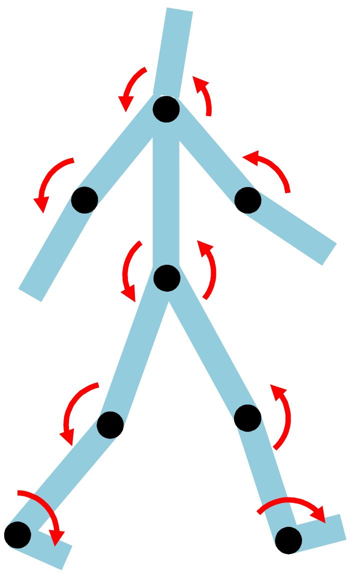


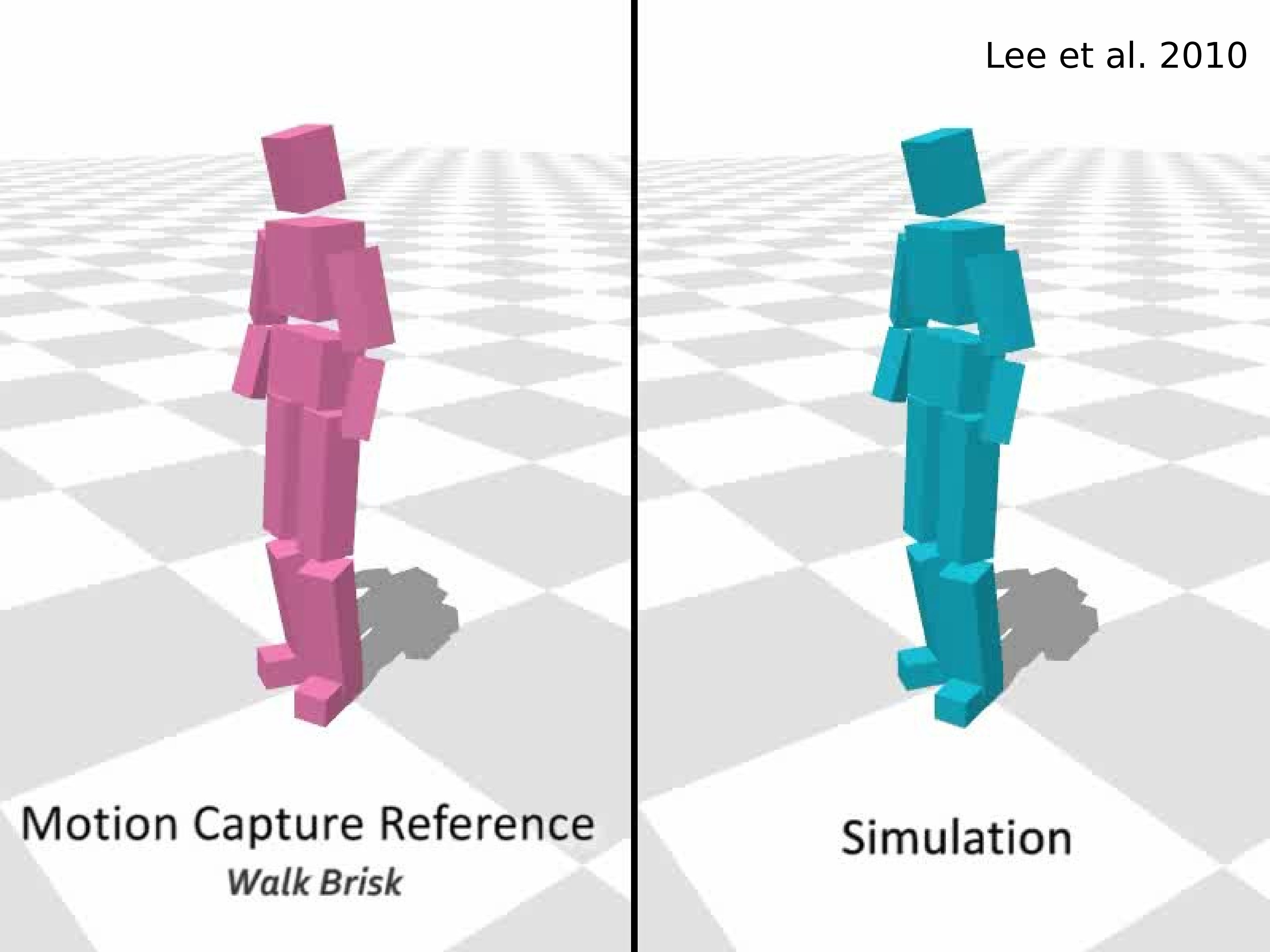

reference motion
simulation

# Biped Control

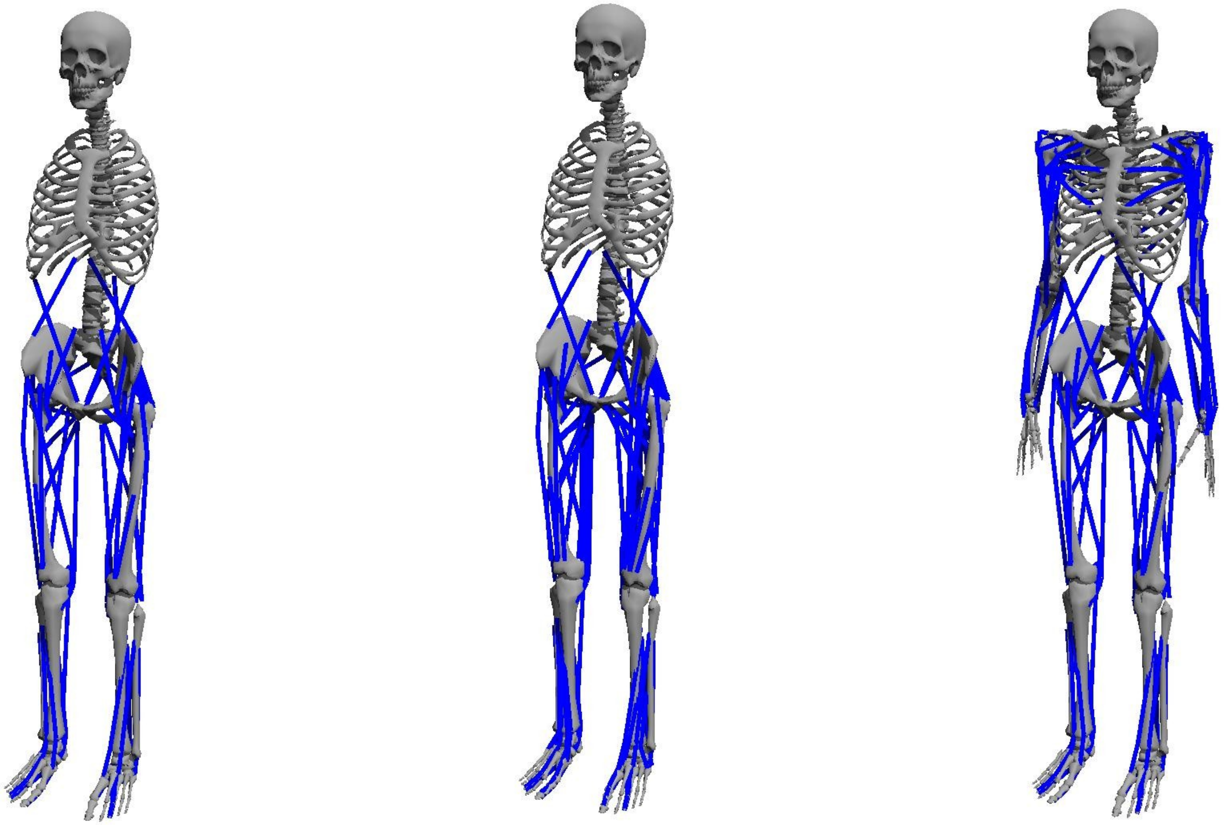
- Under-actuated system
- We can only use internal joint torques



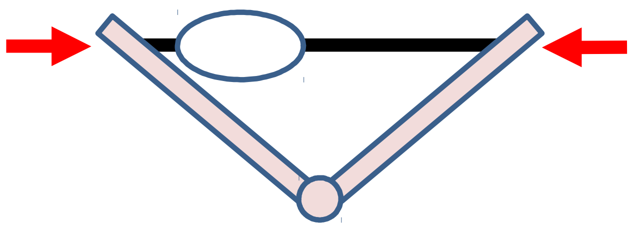



Lee et al. 2010
Motion Capture Reference
Walk Brisk
Simulation

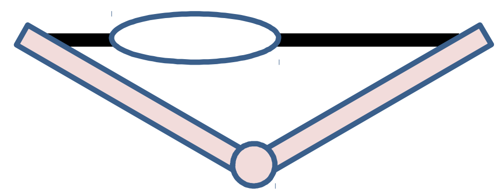
# Musculoskeletal Models



# Control Signal - Muscle Activation

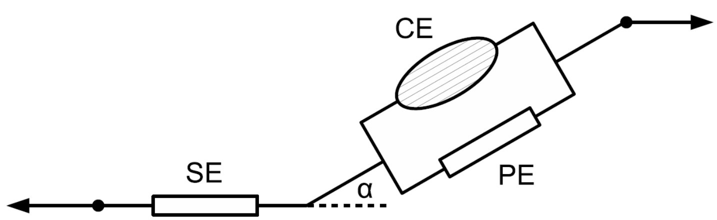


activation=1

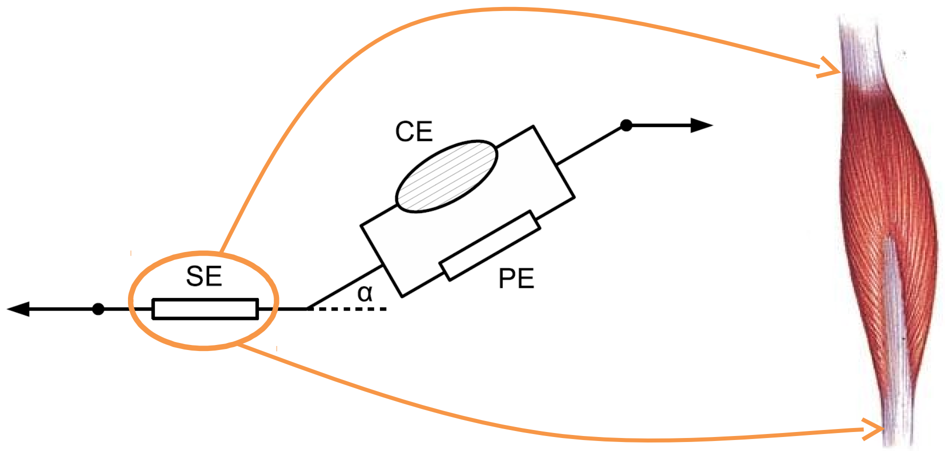


activation=0

# Hill-Type Muscle Model




# Hill-Type Muscle Model



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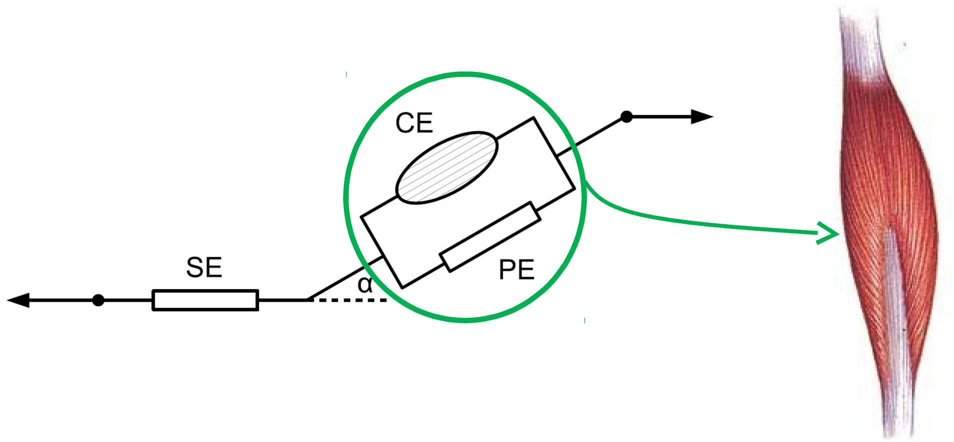

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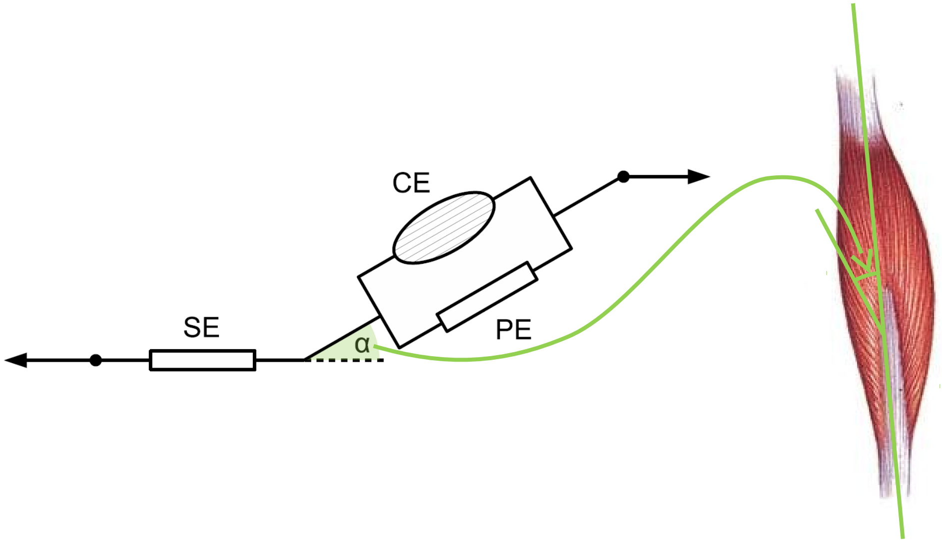
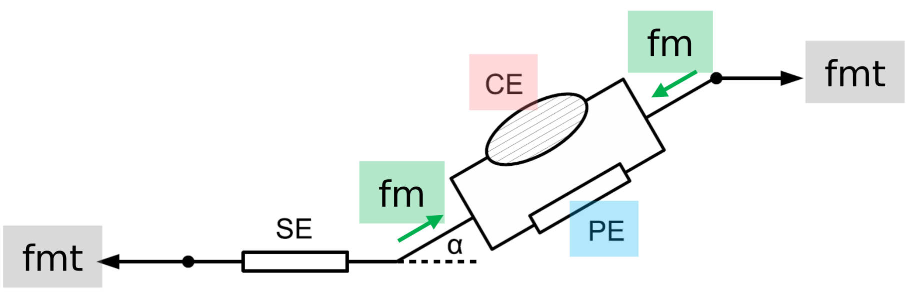
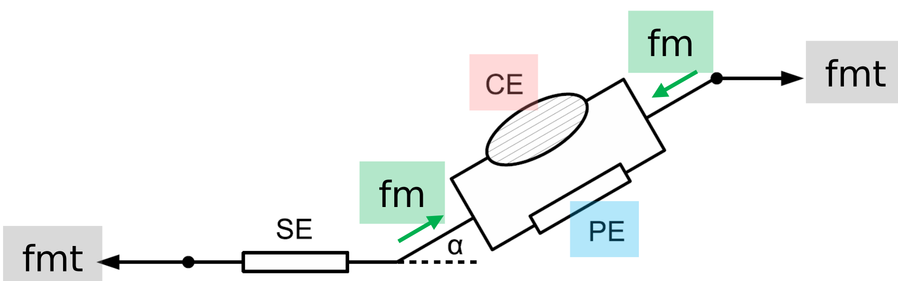


# Muscle Force Generation




$$f_{mt} = f_m \cdot cos(\alpha) = (f_{ce} + f_{pe})cos(\alpha)$$

# Muscle Force Generation

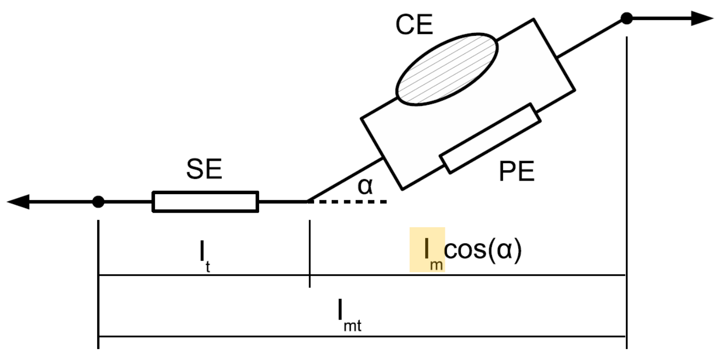



$$f_{mt} = f_m \cdot cos(\alpha) = (f_{ce} + f_{pe})cos(\alpha)$$

$$f_m = a \cdot g_{al}(l_m) \cdot g_v(\dot{l}_m) + g_{pl}(l_m) + b \cdot \dot{l}_m$$

Activation

# Contraction Dynamics




$$f_m = a \cdot g_{al}(l_m) \cdot g_v(\dot{l}_m) + g_{pl}(l_m) + b \cdot \dot{l}_m$$

$$\Rightarrow \quad \dot{l}_m = func(a, l_m)$$

# Thank you