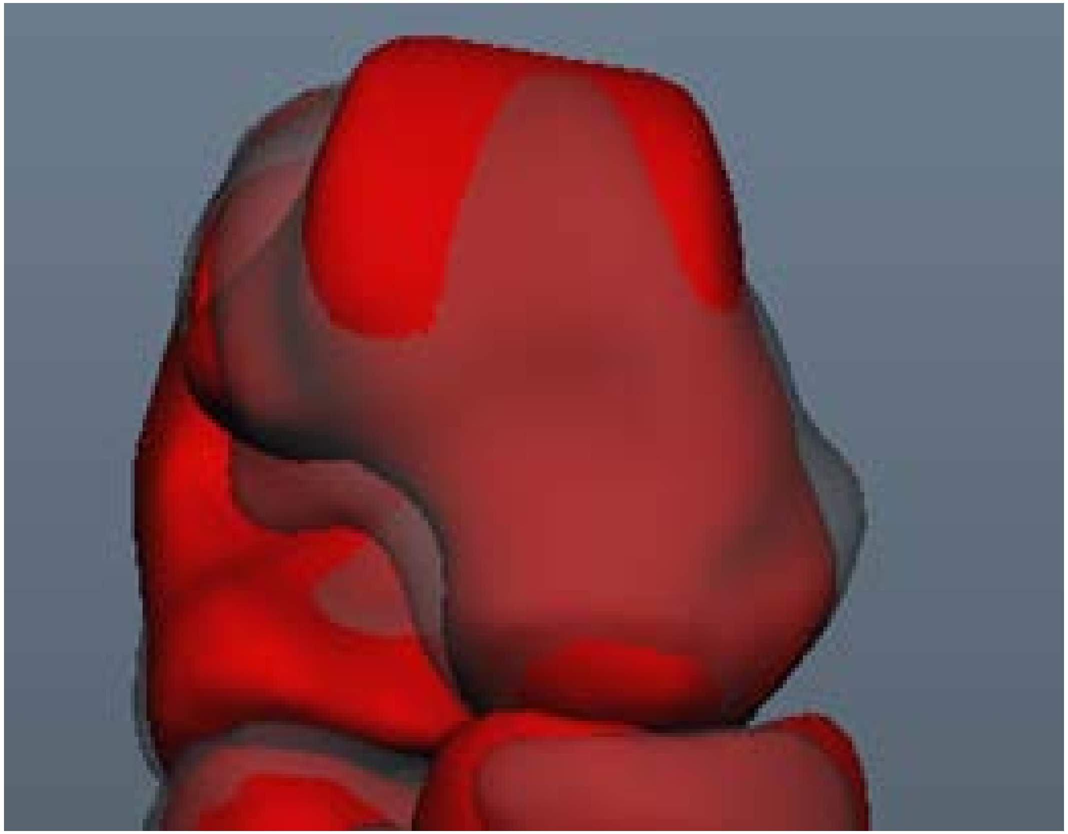
Shape correspondence-> SSM



가까운 점 매칭의 문제점

a)N:1 대응

b) 특징점 보존 안됨

↓

a) 1:1 대응

b) 특징점 보존

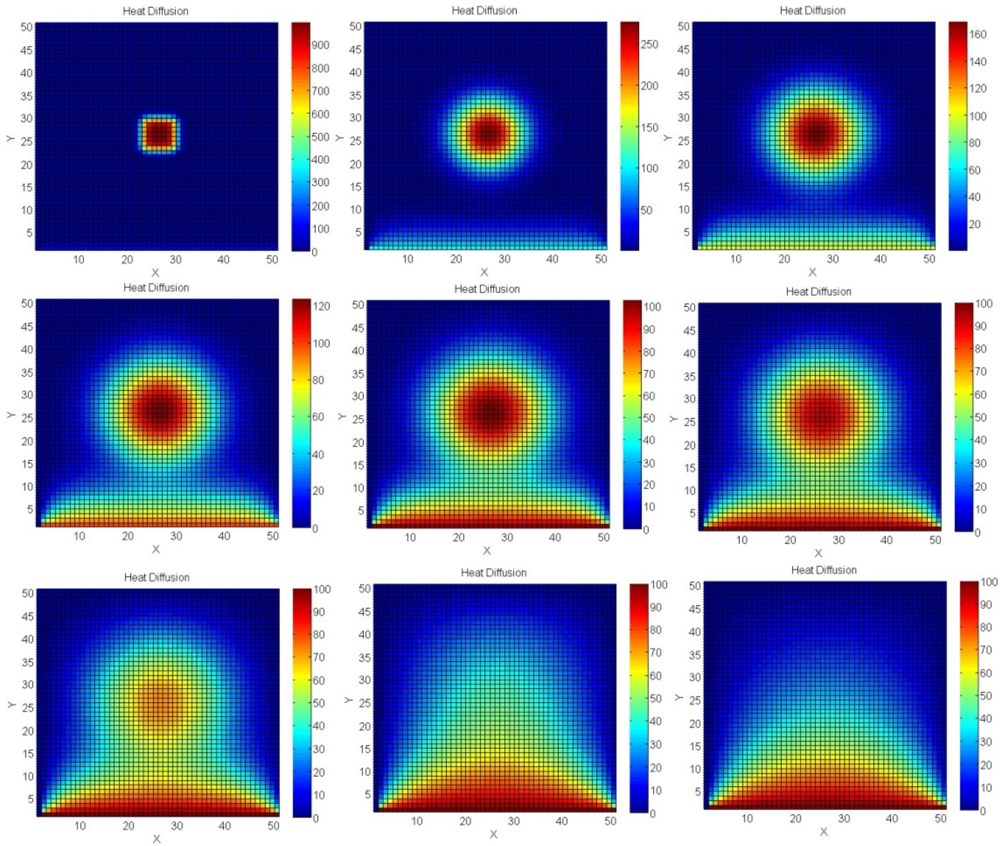
c) 최대한 거리 보존

Heat kernel signature

- 열 전달 미분 방정식을 이용하면
- 짧은 시간 = 지역적으로는 표면의 곡률을 잡아내고
- 긴 시간 = 전역적인 생김새 잡아내게

Heat equation

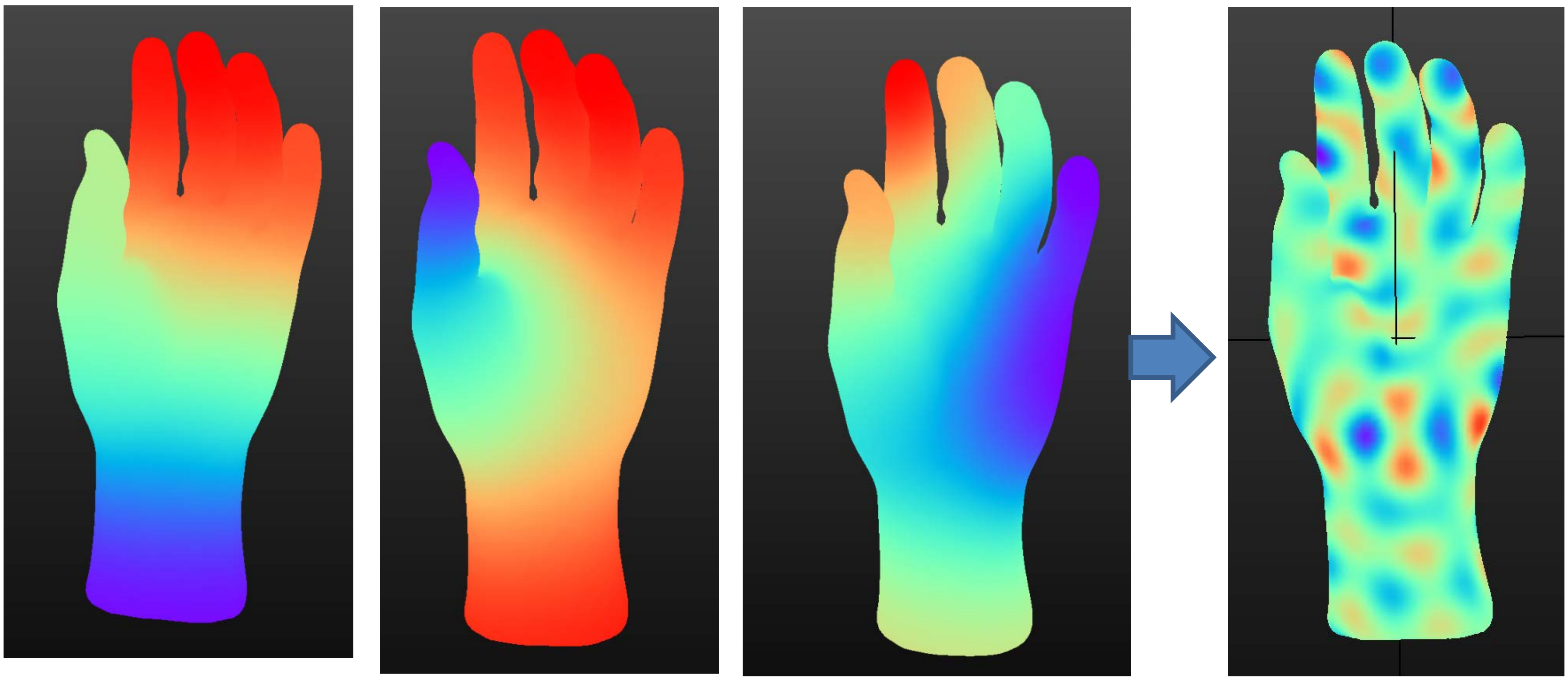
$$\Delta_M u(x,t) = -\frac{\partial u(x,t)}{\partial t},$$



Spectral analysis

- 함수 = 차원이 무한한 벡터로 생각
- 어떤 함수든 기저(basis) 함수의 선형 조합으로 표현 가능.
- Ex) 푸리에 분석(1 차원)

EIGEN FUNCTIONS



1 2 3 200

라플라스 연산자의 기저 함수

$$\Delta\phi_i = \lambda_i\phi_i.$$

기저 함수를 구하면 열 분포 u(x,y) 함수도 기저의 조합으로
표현 가능

$$\Delta_M u(x,t) = -\frac{\partial u(x,t)}{\partial t},$$

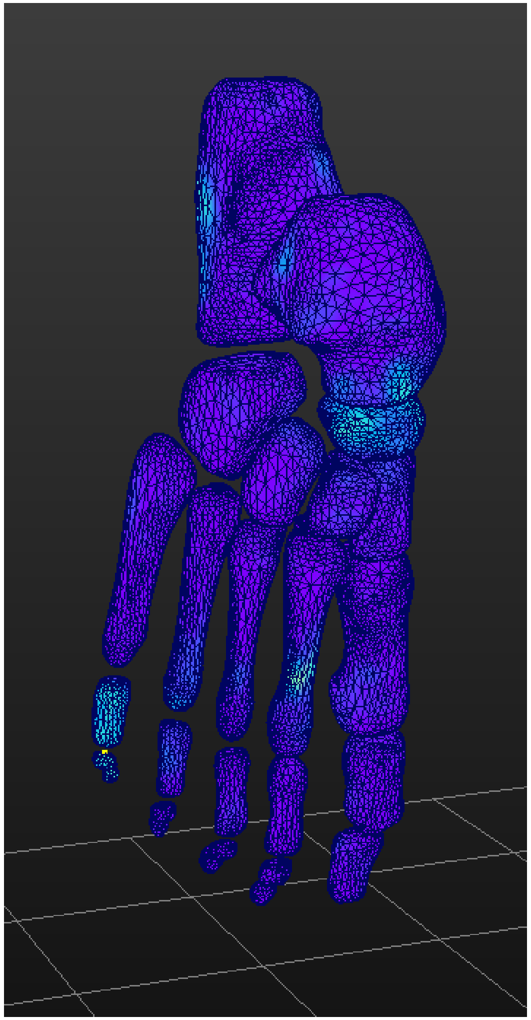
$$\frac{\partial}{\partial t}\langle u(t,\cdot),\phi_i(\cdot)\rangle\phi_i(x) = \langle u(t,\cdot),\phi_i(\cdot)\rangle\lambda_i\phi_i(x)$$

$$\int_M k_t(x,y)f(y)dy \qquad k_t(x,y) = \sum_{i=1}^{\infty} e^{\lambda_i t}\phi_i(x)\phi_i(y)$$

Heat Kernel signature

------------------------------→T increase------------------------------→

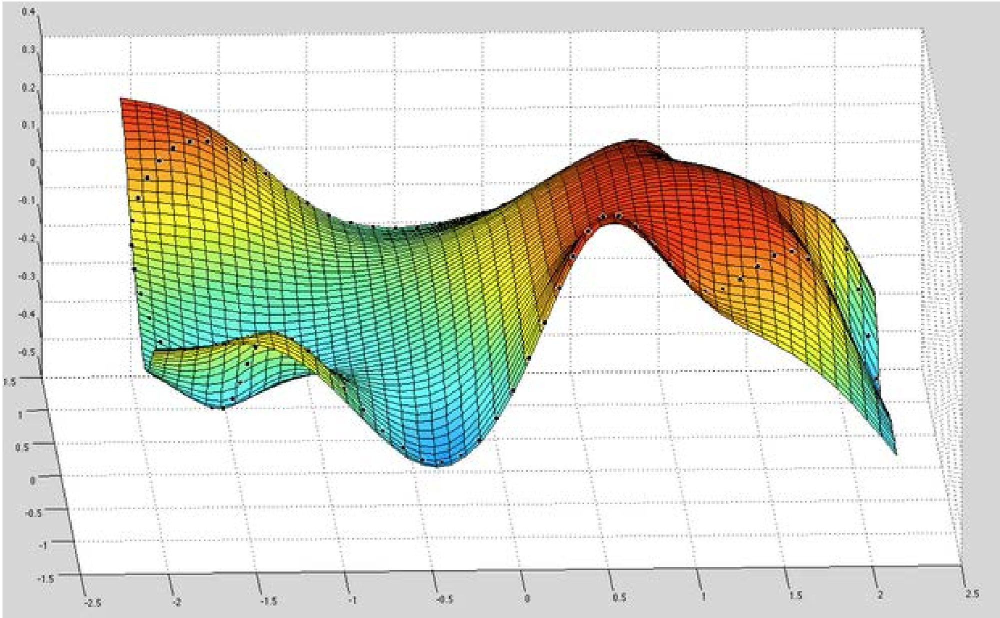
Parameter 찾고 있음..



a)라플라스 연산자 평면에서는

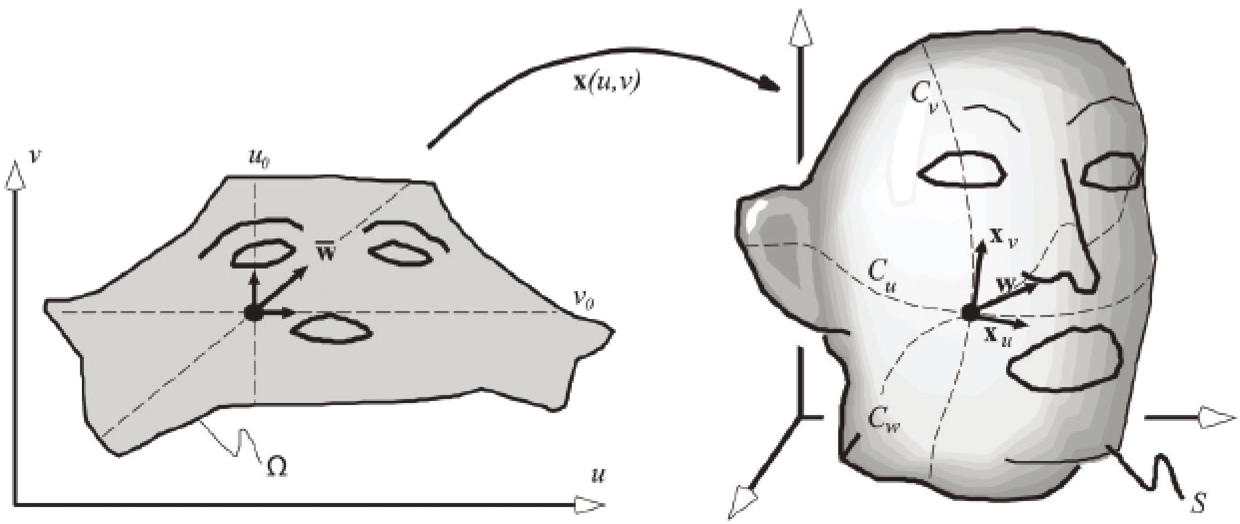
$$\Delta f = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2}$$

b)라플라스 연산자 곡면에서는?

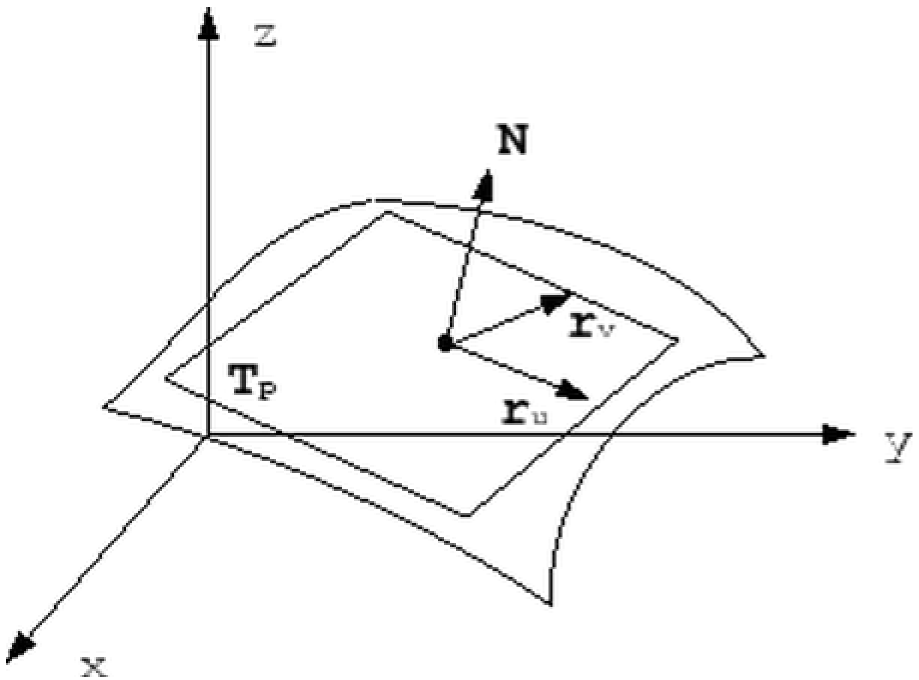


u,v 매개화, 차이 보정

x,y,z = p(u,v)



$$\mathbf{x}_u(u_0, v_0) := \frac{\partial \mathbf{x}}{\partial u}(u_0, v_0) \quad \text{and} \quad \mathbf{x}_v(u_0, v_0) := \frac{\partial \mathbf{x}}{\partial v}(u_0, v_0)$$



- Gradient

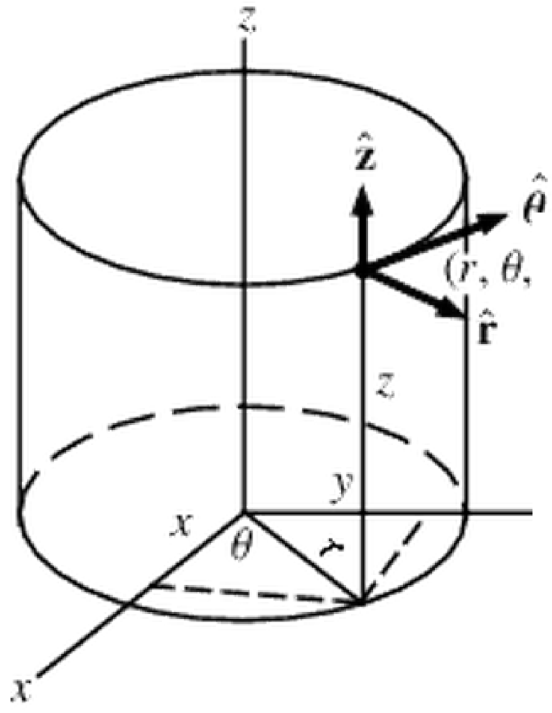
$$\nabla f = g^{ik} \frac{\partial f}{\partial x^k} \frac{\partial}{\partial x^i}.$$

$$g_{i,j} = \left\langle \frac{\partial \varphi}{\partial x_i}, \frac{\partial \varphi}{\partial x_j} \right\rangle$$

- Laplace

$$\begin{aligned} \Delta f &:= div \, \nabla f \\ &= \frac{1}{\sqrt{|det \, g|}} \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} \left(\sqrt{|det \, g|} \cdot g^{i,j} \cdot \frac{\partial}{\partial x_j} f \right) \end{aligned}$$

Example cylinder coordinates



$$\nabla \equiv \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\mathbf{z}}\frac{\partial}{\partial z},$$

$$\iiint_D f(x,y,z)\,dx\,dy\,dz \Rightarrow \iiint_D f(r\cos\theta, r\sin\theta, z)\,r\,dr\,d\theta\,dz$$

Discrete Laplace operator

컴퓨터로 구현할 때는 아래와 같은
sampling을 통한 difference로 대체

F” = f(t+h) + f(t-h) -2* f(t) / h^2

$$\mathrm{L}_K^h f(w) = \frac{1}{4\pi h^2} \sum_{t \in K} \frac{\mathrm{Area}(t)}{\#t} \sum_{p \in V(t)} e^{-\frac{\|p-w\|^2}{4h}} (f(p) - f(w))$$